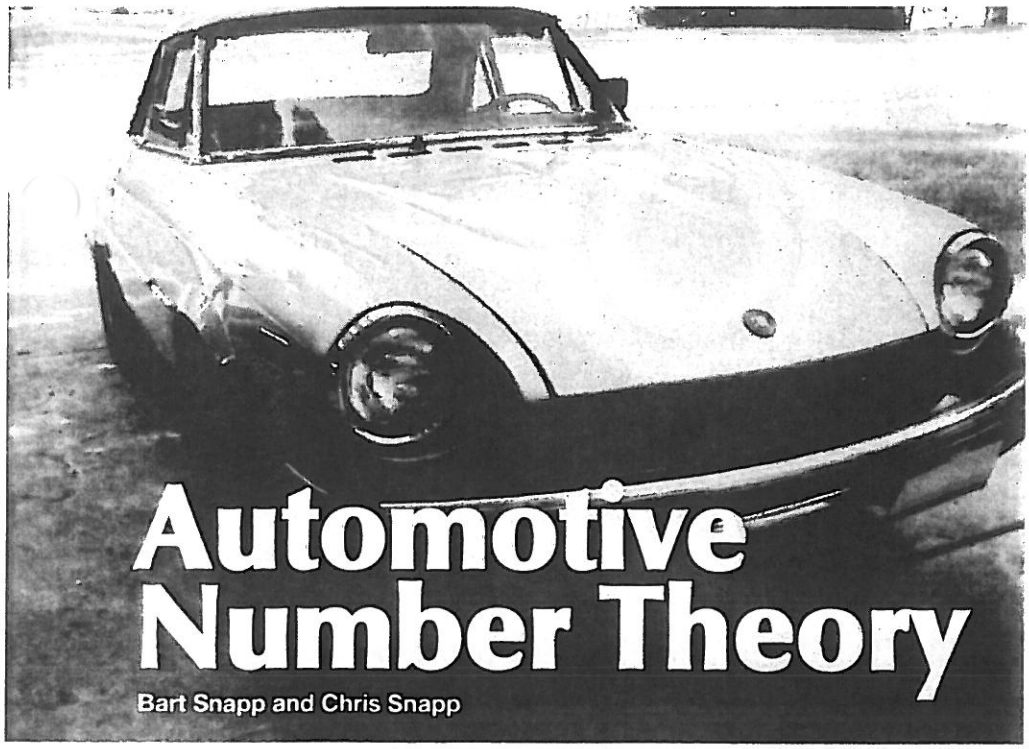

Automotive Number Theory

Bart Snapp and Chris Snapp

Photos courtesy of Doug Snapp

> Why did the Fiat engineers choose the numbers 17 and 21? Were they chosen purposely, or were they an example of haphazard engineering?

Mathematics is everywhere. We were pleasantly reminded of this fact while repairing the ignition switch on a 1981 Fiat Spider. In the course of our work, we removed the steering wheel and the steering column. However, we neglected to mark their original positions. All was fine until it came time to put the parts back together. The steering wheel was no longer centered! The car drove fine, but when driving straight ahead the steering wheel was out of alignment. It was off by a rotation of 5 degrees to the right. This would not do!

After some reflection, we realized that the problem stemmed from our understanding of the design of the steering wheel and column assembly. Specifically, it was the way the steering column connected the steering wheel to the rest of the car that was at the root of the problem. Image 1 shows a picture of the back of the steering wheel.

As you can see, the back of the wheel has 21 notches which match up with the 21 notches on the top of the steering column shown in Image 2.

The steering column then attaches to the car with a 17 notch connector pictured in Image 3. Taken together, there are 21 possible orientations for the steering wheel, and 17 orientations for the steering column for a total of $21 \times 17 = 357$ possible configurations.

Question: What orientation of the steering wheel and steering column lead to a perfectly centered steering wheel?

To answer this question we'll use number theory! Specifically, we will use facts about the greatest common divisor (GCD) and Diophantine equations.

Question: What's a Diophantine equation?

Glad you asked! A *Diophantine equation* is an equation whose unknowns are assumed to be integers. To see the connection between Diophantine equations and this problem, consider the following: We found that there were 21 different orientations (equally spaced) of the steering wheel and 17 different orientations (equally spaced) of the steering column. The upshot is that adjusting the orientation of the wheel by one notch causes a $360/21 \approx 17$ degree shift, while adjusting the orientation of the column by one notch causes about a $360/17 \approx 21$ degree shift. Technically speaking, we are actually not working in degrees; rather, we are working with a unit which is $1/357^{\text{th}}$ of a complete rotation. This is close enough for our purposes.

To adjust the steering wheel and column so that the wheel is turned 1 degree to the right from its original position amounts to solving the Diophantine equation

$$17w + 21c = 1$$

where w represents how many notches to the right we adjust the wheel and c represents how many notches to the right we adjust the column. Now we must point out something important:

A Diophantine equation of the form $ax + by = g$ has a solution if and only if the GCD of a and b divides g.

Since the GCD of 17 and 21 is 1, we know our equation has a solution. How do we find the solution? It turns out that there is an old trick which is related to the Euclidean algorithm. Essentially, in the process of computing the GCD, we are able to find a solution to the equation. In this case we find that $w = 5$ and $c = 4$:

$$(17)(5) - (21)(4) = 1.$$

Because our wheel is off by 5 degrees to the right, we want to turn the wheel 5 degrees to the left. To achieve this we multiply the above equation by -5 to get

$$(17)(-25) + (21)(20) = -5.$$

This means the wheel needs to be turned 25 notches to the left, and the column needs to be adjusted 20

notches to the right. Because there are only 21 different orientations of the wheel, we really only need to adjust the wheel four notches to the left (25 is congruent to 4 modulo 21). Similarly, there are only 17 different orientations of the column, and so we really only need to adjust the column three notches to the right (20 is congruent to 3 modulo 17).

Summing up, we adjusted the wheel four notches to the left and adjusted the column three notches to the right and...Ta daa! The steering wheel was now centered and we were ready for a drive. OK—we are done with that question. However, there are always more questions! In particular:

Question: Why did the Fiat engineers choose the numbers 17 and 21? Were they chosen purposely, or were they an example of haphazard engineering?

Well, apparently, the Fiat engineers knew that they would need to adjust the steering wheel in 1 degree increments. Since the square-root of 360 is about 19, they wanted two integers close to 19 with no common factors other than 1. Since 18 and 20 won't work, they used 17 and 21. Number theory to the rescue again!

Further Reading

If you think this sort of thing is interesting, we encourage you to check out: *Number Theory Through Inquiry* by David C. Marshall, Edward Odell & Michael Starbird.

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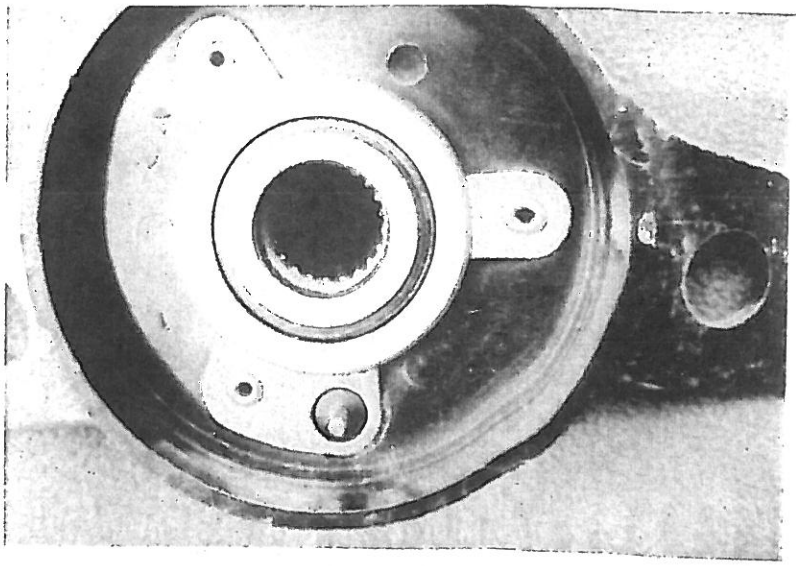

Image 1: The back side of the steering wheel showing the 21 notched hole.

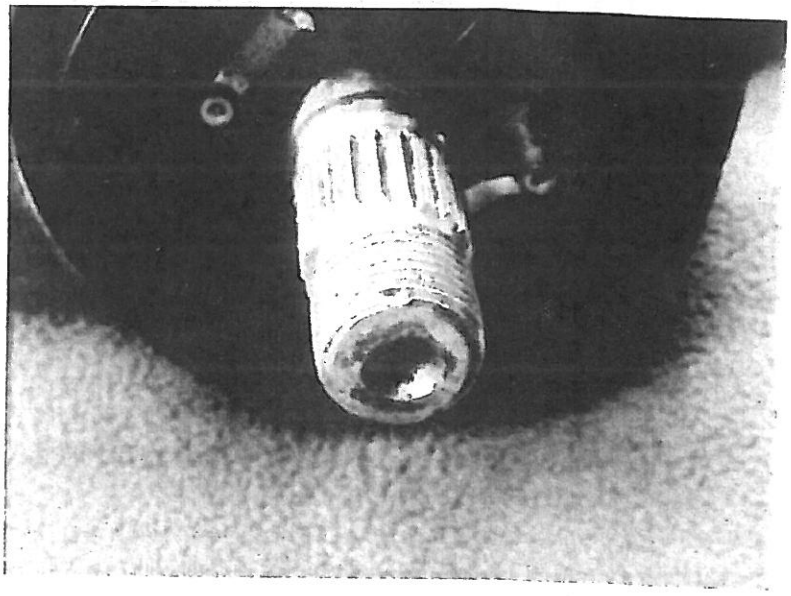

Image 2: The corresponding 21 notches on the steering column where the wheel is attached.

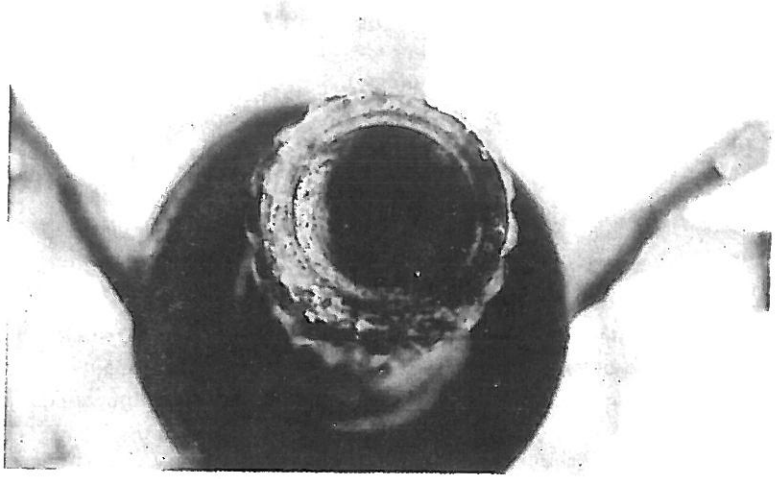

Image 3: The 17 notches at the opposite end of the steering column.